BOLETÍN 02 - RADICALES 02 - MATEMÁTICAS 4.º ESO

1) Calcula el valor de las siguientes raíces cuadradas:

a) $\sqrt{144}$ b) $\sqrt{196}$ c) $\sqrt{225}$ d) $\sqrt{1}$

2) Simplifica extrayendo factores fuera de la raíz:

a) $\sqrt{108}$ b) $\sqrt{147}$ c) $\sqrt{288}$ d) $\sqrt{242}$

3) Calcula y simplifica:

a) $\sqrt{5^2 \cdot 2}$ b) $\sqrt{3^4 \cdot 2}$ c) $\sqrt{\dfrac{98}{2}}$ d) $\sqrt{12 \cdot 3}$

4) Expresa en forma de potencia con exponente fraccionario:

a) $\sqrt[3]{a}$ b) $\sqrt[4]{x^3}$ c) $\sqrt{y^5}$ d) $\sqrt[5]{b^2}$

5) Expresa en forma de raíz:

a) $x^{\frac{2}{3}}$ b) $a^{\frac{3}{2}}$ c) $b^{\frac{5}{4}}$ d) $y^{\frac{4}{5}}$

6) Opera con raíces semejantes:

a) $4\sqrt{3} + 2\sqrt{12} - \sqrt{27}$ b) $3\sqrt{5} - \sqrt{20} + 2\sqrt{45}$

c) $2\sqrt{7} + \sqrt{63} - 3\sqrt{28}$ d) $5\sqrt{2} - \sqrt{18} + 2\sqrt{8}$

7) Multiplica y simplifica:

a) $\sqrt{8} \cdot \sqrt{18}$ b) $3\sqrt{2} \cdot 2\sqrt{14}$ c) $2\sqrt{6} \cdot \sqrt{24}$ d) $4\sqrt{3} \cdot 3\sqrt{12}$

8) Divide y simplifica:

a) $\dfrac{\sqrt{96}}{\sqrt{6}}$ b) $\dfrac{\sqrt{180}}{\sqrt{5}}$ c) $\dfrac{3\sqrt{50}}{5\sqrt{2}}$ d) $\dfrac{4\sqrt{27}}{6\sqrt{3}}$

9) Racionaliza los denominadores:

a) $\dfrac{7}{\sqrt{5}}$ b) $\dfrac{3}{2\sqrt{7}}$ c) $\dfrac{5}{\sqrt{6} - \sqrt{2}}$ d) $\dfrac{2\sqrt{3}}{\sqrt{3} + 1}$

10) Opera y simplifica:

a) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ b) $(\sqrt{6} + 2\sqrt{2})(\sqrt{6} - \sqrt{2})$

c) $(\sqrt{3} + \sqrt{2})^2$ d) $(2\sqrt{5} - \sqrt{2})^2$

BOLETÍN 02 - RADICALES 02 - MATEMÁTICAS 4.º ESO

11) Extrae factores fuera de la raíz:

a) $\sqrt{192x^4y^3}$ b) $\sqrt{108a^5b^2}$ c) $\sqrt{250m^3n^6}$ d) $\sqrt{432p^4q^5}$

12) Calcula las raíces cúbicas:

a) $\sqrt[3]{216}$ b) $\sqrt[3]{-125}$ c) $\sqrt[3]{343}$ d) $\sqrt[3]{1000}$

13) Simplifica los siguientes radicales con variables:

a) $\sqrt{x^6y^4}$ b) $\sqrt{a^4b^5}$ c) $\sqrt{m^7n^2}$ d) $\sqrt{p^2q^9}$

14) Calcula las siguientes expresiones mixtas:

a) $\sqrt{16+9}+\sqrt{4}$ b) $\sqrt{25+11}-\sqrt{9}$ c) $\sqrt{49-13}+\sqrt{16}$ d) $\sqrt{64+17}-\sqrt{81}$

15) Racionaliza y simplifica:

a) $\dfrac{4}{\sqrt{3}+1}$ b) $\dfrac{5}{\sqrt{6}-2}$ c) $\dfrac{3}{\sqrt{5}+\sqrt{2}}$ d) $\dfrac{\sqrt{10}}{\sqrt{10}-\sqrt{5}}$

16) Aplica las propiedades de los radicales para simplificar:

a) $\sqrt{4ab}$ b) $\sqrt{\dfrac{9a}{b}}$ c) $\sqrt{a^4b^3}$ d) $\sqrt{\dfrac{a^5}{b^2}}$

17) Expresa en una sola raíz y simplifica:

a) $\sqrt{8}\cdot\sqrt{27}$ b) $\sqrt{12}\cdot\sqrt{3}$ c) $\sqrt{15}\cdot\sqrt{60}$ d) $\sqrt{14}\cdot\sqrt{21}$

18) Índices mayores: extrae factores y simplifica:

a) $\sqrt[3]{250x^4y^3}$ b) $\sqrt[3]{432a^7b^4}$

c) $\sqrt[4]{162x^5}$ d) $\sqrt[5]{96y^6z^{10}}$

19) Calcula con potencias y radicales:

a) $(\sqrt{7})^4$ b) $(\sqrt[3]{27})^3$ c) $(\sqrt[4]{16})^3$ d) $(\sqrt{2})^5$

20) Problema de aplicación.

Un cuadrado tiene lado $\sqrt{32}$ cm.

a) Calcula su perímetro.

b) Calcula su área.

c) Simplifica ambos resultados.

d) Expresa el lado como potencia con exponente racional.

Soluciones

1)

a) $\sqrt{144} = \boxed{12}$

b) $\sqrt{196} = \boxed{14}$

c) $\sqrt{225} = \boxed{15}$

d) $\sqrt{1} = \boxed{1}$

2)

a) $\sqrt{108} = \sqrt{36 \cdot 3} = \boxed{6\sqrt{3}}$

b) $\sqrt{147} = \sqrt{49 \cdot 3} = \boxed{7\sqrt{3}}$

c) $\sqrt{288} = \sqrt{144 \cdot 2} = \boxed{12\sqrt{2}}$

d) $\sqrt{242} = \sqrt{121 \cdot 2} = \boxed{11\sqrt{2}}$

3)

a) $\sqrt{5^2 \cdot 2} = \sqrt{25 \cdot 2} = \boxed{5\sqrt{2}}$

b) $\sqrt{3^4 \cdot 2} = \sqrt{81 \cdot 2} = \boxed{9\sqrt{2}}$

c) $\sqrt{\dfrac{98}{2}} = \sqrt{49} = \boxed{7}$

d) $\sqrt{12 \cdot 3} = \sqrt{36} = \boxed{6}$

4)

a) $\sqrt[3]{a} = \boxed{a^{\frac{1}{3}}}$

b) $\sqrt[4]{x^3} = \boxed{x^{\frac{3}{4}}}$

c) $\sqrt{y^5} = \boxed{y^{\frac{5}{2}}}$

d) $\sqrt[5]{b^2} = \boxed{b^{\frac{2}{5}}}$

5)

a) $x^{\frac{2}{3}} = \boxed{\sqrt[3]{x^2}}$

b) $a^{\frac{3}{2}} = \sqrt{a^3} = \boxed{a\sqrt{a}}$

c) $b^{\frac{5}{4}} = \sqrt[4]{b^5} = \boxed{b\sqrt[4]{b}}$

d) $y^{\frac{4}{5}} = \boxed{\sqrt[5]{y^4}}$

6)

a) $4\sqrt{3} + 2\sqrt{12} - \sqrt{27} = 4\sqrt{3} + 2\sqrt{4 \cdot 3} - \sqrt{9 \cdot 3} = 4\sqrt{3} + 4\sqrt{3} - 3\sqrt{3} = \boxed{5\sqrt{3}}$

b) $3\sqrt{5} - \sqrt{20} + 2\sqrt{45} = 3\sqrt{5} - \sqrt{4 \cdot 5} + 2\sqrt{9 \cdot 5} = 3\sqrt{5} - 2\sqrt{5} + 6\sqrt{5} = \boxed{7\sqrt{5}}$

c) $2\sqrt{7} + \sqrt{63} - 3\sqrt{28} = 2\sqrt{7} + \sqrt{9 \cdot 7} - 3\sqrt{4 \cdot 7} = 2\sqrt{7} + 3\sqrt{7} - 6\sqrt{7} = \boxed{-\sqrt{7}}$

d) $5\sqrt{2} - \sqrt{18} + 2\sqrt{8} = 5\sqrt{2} - \sqrt{9 \cdot 2} + 2\sqrt{4 \cdot 2} = 5\sqrt{2} - 3\sqrt{2} + 4\sqrt{2} = \boxed{6\sqrt{2}}$

7)

a) $\sqrt{8} \cdot \sqrt{18} = \sqrt{144} = \boxed{12}$

b) $3\sqrt{2} \cdot 2\sqrt{14} = 6\sqrt{28} = 6\sqrt{4 \cdot 7} = \boxed{12\sqrt{7}}$

c) $2\sqrt{6} \cdot \sqrt{24} = 2\sqrt{144} = \boxed{24}$

d) $4\sqrt{3} \cdot 3\sqrt{12} = 12\sqrt{36} = \boxed{72}$

8)

a) $\dfrac{\sqrt{96}}{\sqrt{6}} = \sqrt{\dfrac{96}{6}} = \boxed{4}$

b) $\dfrac{\sqrt{180}}{\sqrt{5}} = \sqrt{\dfrac{180}{5}} = \boxed{6}$

c) $\dfrac{3\sqrt{50}}{5\sqrt{2}} = \dfrac{3}{5}\sqrt{\dfrac{50}{2}} = \dfrac{3}{5}\sqrt{25} = \boxed{3}$

d) $\dfrac{4\sqrt{27}}{6\sqrt{3}} = \dfrac{2}{3}\sqrt{\dfrac{27}{3}} = \dfrac{2}{3}\sqrt{9} = \boxed{2}$

9)

a) $\dfrac{7}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{7\sqrt{5}}{(\sqrt{5})^2} = \boxed{\dfrac{7\sqrt{5}}{5}}$

b) $\dfrac{3}{2\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{3\sqrt{7}}{2(\sqrt{7})^2} = \boxed{\dfrac{3\sqrt{7}}{14}}$

c) $\dfrac{5}{\sqrt{6}-\sqrt{2}} \cdot \dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \dfrac{5(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} = \dfrac{5(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$
$= \dfrac{5(\sqrt{6}+\sqrt{2})}{6-2} = \boxed{\dfrac{5(\sqrt{6}+\sqrt{2})}{4}}$

d) $\dfrac{2\sqrt{3}}{\sqrt{3}+1} \cdot \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{2\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \dfrac{6-2\sqrt{3}}{(\sqrt{3})^2-1^2}$
$= \dfrac{6-2\sqrt{3}}{3-1} = \boxed{3-\sqrt{3}}$

10)

a) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = (\sqrt{5})^2-(\sqrt{3})^2 = 5-3 = \boxed{2}$

b) $(\sqrt{6}+2\sqrt{2})(\sqrt{6}-\sqrt{2}) = (\sqrt{6})^2-\sqrt{6}\sqrt{2}+2\sqrt{2}\sqrt{6}-2(\sqrt{2})^2$
$= 6-\sqrt{12}+2\sqrt{12}-4 = 2+\sqrt{12} = \boxed{2+2\sqrt{3}}$

c) $(\sqrt{3}+\sqrt{2})^2 = (\sqrt{3})^2+2\sqrt{3}\sqrt{2}+(\sqrt{2})^2$
$= 3+2\sqrt{6}+2 = \boxed{5+2\sqrt{6}}$

d) $(2\sqrt{5}-\sqrt{2})^2 = (2\sqrt{5})^2-2\cdot 2\sqrt{5}\cdot\sqrt{2}+(\sqrt{2})^2$
$= 20-4\sqrt{10}+2 = \boxed{22-4\sqrt{10}}$

11)

a) $\sqrt{192x^4y^3} = \sqrt{64\cdot 3\cdot x^4y^2\cdot y} = \boxed{8x^2y\sqrt{3y}}$

b) $\sqrt{108a^5b^2} = \sqrt{36\cdot 3\cdot a^4a\cdot b^2} = \boxed{6a^2b\sqrt{3a}}$

c) $\sqrt{250m^3n^6} = \sqrt{25\cdot 10\cdot m^2m\cdot n^6} = \boxed{5mn^3\sqrt{10m}}$

d) $\sqrt{432p^4q^5} = \sqrt{144\cdot 3\cdot p^4q^4\cdot q} = \boxed{12p^2q^2\sqrt{3q}}$

12)

a) $\sqrt[3]{216} = \boxed{6}$

b) $\sqrt[3]{-125} = \boxed{-5}$

c) $\sqrt[3]{343} = \boxed{7}$

d) $\sqrt[3]{1000} = \boxed{10}$

13)

a) $\sqrt{x^6y^4} = \boxed{x^3y^2}$

b) $\sqrt{a^4b^5} = \sqrt{a^4b^4 \cdot b} = \boxed{a^2b^2\sqrt{b}}$

c) $\sqrt{m^7n^2} = \sqrt{m^6m \cdot n^2} = \boxed{m^3n\sqrt{m}}$

d) $\sqrt{p^2q^9} = \sqrt{p^2q^8 \cdot q} = \boxed{pq^4\sqrt{q}}$

14)

a) $\sqrt{16+9} + \sqrt{4} = 5 + 2 = \boxed{7}$

b) $\sqrt{25+11} - \sqrt{9} = 6 - 3 = \boxed{3}$

c) $\sqrt{49-13} + \sqrt{16} = 6 + 4 = \boxed{10}$

d) $\sqrt{64+17} - \sqrt{81} = 9 - 9 = \boxed{0}$

15)

a) $\dfrac{4}{\sqrt{3}+1}\cdot\dfrac{\sqrt{3}-1}{\sqrt{3}-1}=\dfrac{4(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\dfrac{4(\sqrt{3}-1)}{(\sqrt{3})^2-1^2}$

$=\dfrac{4(\sqrt{3}-1)}{3-1}=\boxed{2\sqrt{3}-2}$

b) $\dfrac{5}{\sqrt{6}-2}\cdot\dfrac{\sqrt{6}+2}{\sqrt{6}+2}=\dfrac{5(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)}=\dfrac{5(\sqrt{6}+2)}{(\sqrt{6})^2-2^2}$

$=\dfrac{5(\sqrt{6}+2)}{6-4}=\boxed{\dfrac{5(\sqrt{6}+2)}{2}}$

c) $\dfrac{3}{\sqrt{5}+\sqrt{2}}\cdot\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\dfrac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$

$=\dfrac{3(\sqrt{5}-\sqrt{2})}{5-2}=\boxed{\sqrt{5}-\sqrt{2}}$

d) $\dfrac{\sqrt{10}}{\sqrt{10}-\sqrt{5}}\cdot\dfrac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}=\dfrac{\sqrt{10}(\sqrt{10}+\sqrt{5})}{(\sqrt{10}-\sqrt{5})(\sqrt{10}+\sqrt{5})}=\dfrac{10+\sqrt{50}}{(\sqrt{10})^2-(\sqrt{5})^2}$

$=\dfrac{10+5\sqrt{2}}{10-5}=\boxed{2+\sqrt{2}}$

16)

a) $\sqrt{4ab}=\sqrt{4}\sqrt{a}\sqrt{b}=\boxed{2\sqrt{a}\,\sqrt{b}}$

b) $\sqrt{\dfrac{9a}{b}}=\dfrac{\sqrt{9}\sqrt{a}}{\sqrt{b}}=\boxed{\dfrac{3\sqrt{a}}{\sqrt{b}}}$

c) $\sqrt{a^4b^3}=\sqrt{a^4b^2\cdot b}=\boxed{a^2b\sqrt{b}}$

d) $\sqrt{\dfrac{a^5}{b^2}}=\sqrt{\dfrac{a^4a}{b^2}}=\boxed{\dfrac{a^2\sqrt{a}}{b}}$

17)

a) $\sqrt{8}\cdot\sqrt{27}=\sqrt{216}=\sqrt{36\cdot 6}=\boxed{6\sqrt{6}}$

b) $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=\boxed{6}$

c) $\sqrt{15}\cdot\sqrt{60}=\sqrt{900}=\boxed{30}$

d) $\sqrt{14}\cdot\sqrt{21}=\sqrt{294}=\sqrt{49\cdot 6}=\boxed{7\sqrt{6}}$

18)

a) $\sqrt[3]{250x^4y^3} = \sqrt[3]{125x^3y^3 \cdot 2x} = \boxed{5xy\sqrt[3]{2x}}$

b) $\sqrt[3]{432a^7b^4} = \sqrt[3]{216a^6b^3 \cdot 2ab} = \boxed{6a^2b\sqrt[3]{2ab}}$

c) $\sqrt[4]{162x^5} = \sqrt[4]{81x^4 \cdot 2x} = \boxed{3x\sqrt[4]{2x}}$

d) $\sqrt[5]{96y^6z^{10}} = \sqrt[5]{32y^5z^{10} \cdot 3y} = \boxed{2yz^2\sqrt[5]{3y}}$

19)

a) $(\sqrt{7})^4 = \left((\sqrt{7})^2\right)^2 = 7^2 = \boxed{49}$

b) $(\sqrt[3]{27})^3 = 3^3 = \boxed{27}$

c) $(\sqrt[4]{16})^3 = 2^3 = \boxed{8}$

d) $(\sqrt{2})^5 = (\sqrt{2})^4\sqrt{2} = 4\sqrt{2} = \boxed{4\sqrt{2}}$

20)

a) $P = 4\sqrt{32} = 4\sqrt{16 \cdot 2} = \boxed{16\sqrt{2}\text{ cm}}$

b) $A = (\sqrt{32})^2 = \boxed{32\text{ cm}^2}$

c) $\boxed{P = 16\sqrt{2}\text{ cm}, \qquad A = 32\text{ cm}^2}$

d) $\sqrt{32} = \boxed{32^{\frac{1}{2}}}$